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Parameters dominating swirl effects on turbulent transport derived from stress–scalar-flux transport equation

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Abstract—Parameters dominating swirl effects on turbulent transport in swirling flow are derived, focusing on the generation terms appearing in stress–scalar-flux transport equations. $\overline{vw}$ and $\partial(rW)/\partial r$ are the factors which determine the characteristics of swirl effects on stress $\overline{uv}$ and scalar flux $\overline{vc}$. In the case of $\partial(rW)/\partial r \leqslant 0$ and $\overline{vw} > 0$, swirl has the effect of promoting turbulent transport, whereas in the case of $\partial(rW)/\partial r > 0$ and $\overline{vw} \leqslant 0$, it has the effect of suppression. In the case of $\partial(rW)/\partial r > 0$ and $\overline{vw} > 0$, swirl can exert either of these effects.

1. INTRODUCTION

Swirling flow is extensively applied in many practical engineering applications such as combustors, fluid machinery, and diesel engines. Swirl is often used to enhance mixing in many furnaces and combustors, whereas in radiant tube combustors, turbulent mixing is retarded by swirl. Swirl is known to have complicated and complex effects on turbulent transport, and it is important to understand the mechanism of the swirl effect on turbulent transport.

Previous studies [1–10] on turbulent swirling flow show that velocity and scalar profiles are markedly affected by swirl as compared with those under non-swirling flow conditions. These swirling flow fields [1–10] are simplified and fundamental ones occur in boundary-layer-type flows where momentum and scalar radial turbulent transport are dominant. The characteristic features due to swirl studied in these flow fields are (1) laminarization phenomena and heat transfer deterioration in a pipe rotating around its longitudinal axis [1–4], (2) retardation of turbulent mixing and of momentum transport by swirling motion imparted in a stationary pipe [5–7], and (3) promotion of turbulent momentum and heat transport by swirl driven by a rotating inner cylinder in an annulus [8–10]. Phenomena (1) and (2) are instances of suppression of turbulent transport due to swirl, whereas (3) is promotion.

We studied turbulent swirling flows (1), (2) and (3) above in order to establish a prediction procedure and to elucidate the effects of swirl on turbulent transport [11–15]. Those studies demonstrated that the characteristic features due to swirl could be predicted by transport equation models of stress [16] and scalar flux [17] (hereafter stress–flux equations) and that rational explanations of these phenomena related to swirl could be made by considering the effects of swirl on generation terms in the stress–flux equations [11–15]. The suppression or the promotion of turbulent transport due to swirl could be interpreted reasonably by estimating swirl effects on the generation terms.

These interpretations are obtained in different flow field geometries. The present paper develops these interpretations in order to establish general and unified principles on the swirl effects independent of flow field geometry. The dominating parameters which discriminate between suppression and promotion of turbulent transport due to swirl are derived from detailed consideration of the generation terms in the stress–flux equations.

2. DERIVATION OF PARAMETERS AND DISCUSSIONS

2.1. *Derivation of parameters*

Swirling flow dealt with in the present study is an axisymmetric boundary-layer-type where fluid density is constant or density fluctuation is negligibly small. Momentum and scalar radial turbulent transport is dominant in the flow fields, and the conservation equations of axial momentum and scalar are shown, respectively, by

$$\frac{DU}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\partial}{r\,\partial r}(r\overline{uv}) \tag{1}$$

$$\frac{DC}{Dt} = -\frac{\partial}{r\,\partial r}(r\overline{vc}) \tag{2}$$

where x and r are the coordinates in the axial and

NOMENCLATURE

D	diffusion term in stress–flux equation
E	dissipation term
p	time-mean pressure
G	generation term in stress–flux equation
r	coordinate in radial direction
R	pressure–strain-rate or pressure–scalar-gradient term in stress–flux equation
U, V, W	time-mean velocity components in axial, radial and tangential directions, respectively
U_i	time-mean velocity components in x_i direction
u_i	fluctuating velocity components in x_i direction
u, v, w	fluctuating velocity components in axial, radial and tangential directions, respectively
x	coordinate in axial direction
x_i	Cartesian coordinate in direction i.

Greek symbol

ρ	fluid density.

Superscript

$^-$	time average.

radial directions, respectively; U and u are the time-mean and fluctuating velocity components in the axial direction, respectively; v is the fluctuating velocity component in the radial direction; C and c are the time-mean and fluctuating scalars, respectively; ρ is the fluid density, and p the time-mean pressure. $(^-)$ denotes conventional time average.

For a transport equation of the correlation $\overline{u_iu_j}$ expressed in Cartesian coordinates, $\overline{u_iu_j}$ is produced by the production term $-\overline{u_iu_k}\,\partial U_j/\partial x_k-\overline{u_ju_k}\,\partial U_i/\partial x_k$. Here, x_i is the Cartesian coordinate in direction i. U_i and u_i are x_i components of time-mean and fluctuating velocities, respectively. In the case of swirling flow expressed in cylindrical coordinates, sourcelike terms arise in the convection term $\mathrm{D}\overline{u_iu_j}/\mathrm{D}t$. If the sourcelike terms are transposed to the right-hand side of the transport equation, they represent a characteristic of production. We define a generation term G as a summation of $-\overline{u_iu_k}\,\partial U_j/\partial x_k-\overline{u_ju_k}\,\partial U_i/\partial x_k$ and sourcelike terms in the convection terms. For example, the transport equation of $\overline{v^2}$ is expressed as follows:

$$U\frac{\partial\overline{v^2}}{\partial x}+V\frac{\partial\overline{v^2}}{\partial r}-2\overline{vw}\frac{W}{r}=2\overline{vw}\frac{W}{r}+R\overline{v^2}+D\overline{v^2}+E\overline{v^2} \quad (3)$$

The generation term $G\overline{v^2}$ is expressed as

$$G\overline{v^2}=4\overline{vw}\frac{W}{r}. \quad (4)$$

Here, W and w are the time-mean and fluctuating velocity components in the tangential direction, respectively. R, D and E represent pressure–strain-rate correlation, diffusion and dissipation terms, respectively. R also represents the pressure–scalar-gradient correlation term in the transport equation of $\overline{vc}$ [equation (6)].

Momentum and scalar transport by turbulent motion, represented by correlations $\overline{uv}$ and $\overline{vc}$ in equations (1) and (2), can be evaluated from the following transport equations:

$$U\frac{\partial\overline{uv}}{\partial x}+V\frac{\partial\overline{uv}}{\partial r}=\underbrace{-\overline{v^2}\frac{\partial U}{\partial r}}_{G\overline{uv}1}\underbrace{+2\overline{uw}\frac{W}{r}}_{G\overline{uv}2}+R\overline{uv}+D\overline{uv} \quad (5)$$

$$U\frac{\partial\overline{vc}}{\partial x}+V\frac{\partial\overline{vc}}{\partial r}=\underbrace{-\overline{v^2}\frac{\partial C}{\partial r}}_{G\overline{vc}1}\underbrace{+2\overline{wc}\frac{W}{r}}_{G\overline{vc}2}+R\overline{vc}+D\overline{vc}. \quad (6)$$

Generation terms $G\overline{uv}1$, $G\overline{uv}2$, $G\overline{vc}1$ and $G\overline{vc}2$ are those shown in equations (5) and (6). $G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$ and $G\overline{vc}1(=-\overline{v^2}\,\partial C/\partial r)$ are gradient-type generation terms which involve velocity and scalar gradients, respectively. $G\overline{uv}2(=2\overline{uw}W/r)$ and $G\overline{vc}2(=2\overline{wc}W/r)$ are the additional generation terms including swirl velocity W, and have large effects on $\overline{uv}$ [11–13, 15] and $\overline{vc}$ [12, 13, 15] in swirling flows.

The previous studies of three types of turbulent swirling flows by the authors revealed that (1) the significant change introduced by swirl in the profiles of U and C could be predicted by the stress–flux equation models [11–13, 15], and (2) turbulent transport of heat, mass and momentum in swirling flows could be explained rationally by considering the effects of swirl on the generation terms $G\overline{uv}1$, $G\overline{uv}2$, $G\overline{vc}1$ and $G\overline{vc}2$. Results obtained for the three types of swirling flows are explained briefly in (a)–(c) below.

(a) Heat transfer deteriorates [4], laminarization phenomena [1–3] and decrease in the wall-friction factor are observed when swirl is driven by a pipe rotating around its axis. The dominating factors which cause these phenomena are the generation terms $G\overline{uv}2(=2\overline{uw}W/r)$ in $\overline{uv}$ equation (5) [11] and $G\overline{vc}2(=2\overline{wc}W/r)$ in $\overline{vc}$ equation (6) [12]. They exert the effects of decreasing $\overline{uv}$ and $\overline{vc}$ due to swirl.

(b) When swirl is imparted to a flow in a stationary pipe, momentum and scalar turbulent transport is suppressed due to swirl where the characteristic axial velocity profile and the retardation of turbulent mixing are observed [5–7]. The dominant contribution of swirl motion to $\overline{uv}$ and $\overline{vc}$ is as follows. The swirl velocity W generates $\overline{uw}$ and $\overline{wc}$. The generation terms $G\overline{uv}2(=2\overline{uw}W/r)$ in $\overline{uv}$ equation (5) and $G\overline{vc}2(=2\overline{wc}W/r)$ in $\overline{vc}$ equation (6) have opposite signs to the other gradient-type generation terms $G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$ and $G\overline{vc}1(=-\overline{v^2}\,\partial C/\partial r)$, respectively. $G\overline{uv}2(=2\overline{uw}W/r)$ and $G\overline{vc}2(=2\overline{wc}W/r)$ suppresses $\overline{uv}$ and $\overline{vc}$, respectively [13]. Here, the effect of suppression means the effect counter to driving gradient diffusion.

(c) When swirl is induced by an inner cylinder rotation in a concentric annulus, the wall friction factor and heat transfer rate increase [8–10, 14]. The dominating contribution of swirl velocity W to $\overline{uv}$ and $\overline{vc}$ is as follows. Swirl velocity W generates positive $\overline{vw}$ which increases $\overline{v^2}$ due to the increase of the positive generation term $G\overline{v^2}(=4\overline{vw}W/r)$ in the $\overline{v^2}$ equation. The increase of $\overline{v^2}$ induces the increase of generation terms $G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$ in $\overline{uv}$ equation (5) and $G\overline{vc}1(=-\overline{v^2}\,\partial C/\partial r)$ in $\overline{vc}$ equation (6), which increase the wall friction and heat transfer [15], respectively. The other generation terms $G\overline{uv}2(=2\overline{uw}W/r)$ and $G\overline{vc}2(=2\overline{wc}W/r)$ also have the effects of promotion, but these are small compared with $G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$ and $G\overline{vc}1(=-\overline{v^2}\,\partial C/\partial r)$, respectively.

Swirl has the effect of suppressing flow fields (a) and (b), whereas it has the effect of promotion in flow field (c). In the present work we derive the general and unified parameters which dominate the swirl effects, i.e. suppression or promotion, in turbulent transport independent of flow field geometry.

The swirl effects above in (a), (b) and (c) can be explained by the characteristics of the generation terms of the turbulent flux equations. Therefore, the principles are constructed focusing on the relation between swirling flow and the generation terms.

The present principles are developed on the basis of the following two assumptions:

(1) A turbulent flux has the same sign as its generation term. This is applicable to equilibrium or nearly equilibrium flows.

(2) When suppression of turbulent transport due to swirl is significant, momentum and scalar countergradient diffusion might occur. The parameters derived are applicable to an extent where swirl effects are not significant enough to induce a countergradient diffusion. These conditions can be expressed as

$$\overline{uv}(-\partial U/\partial r) > 0 \tag{7}$$

$$\overline{vc}(-\partial C/\partial r) > 0. \tag{8}$$

Conditions under which a countergradient diffusion is induced are noted in Section 2.3.

$\overline{uw}$ is included in $G\overline{uv}2(=2\overline{uw}W/r)$ and the generation term of $\overline{uw}$ ($G\overline{uw}$) is shown in the following equation:

$$G\overline{uw} = \underbrace{-\overline{vw}\frac{\partial U}{\partial r}}_{G\overline{uw}1}\ \underbrace{-\overline{uv}\frac{\partial(rW)}{r\,\partial r}}_{G\overline{uw}2}. \tag{9}$$

$\overline{wc}$ is included in $G\overline{vc}2(=2\overline{wc}W/r)$ and its generation term is presented,

$$G\overline{wc} = \underbrace{-\overline{vw}\frac{\partial C}{\partial r}}_{G\overline{wc}1}\ \underbrace{-\overline{vc}\frac{\partial(rW)}{r\,\partial r}}_{G\overline{wc}2}. \tag{10}$$

$G\overline{uw}1$, $G\overline{uw}2$, $G\overline{wc}1$ and $G\overline{wc}2$ are shown in equations (9) and (10). $\overline{v^2}$, $\overline{uw}$ and $\overline{wc}$ are included in the generation terms of $\overline{uv}$ and $\overline{vc}$ [$G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$, $G\overline{uv}2(=2\overline{uw}W/r)$, $G\overline{vc}1(=-\overline{v^2}\,\partial C/\partial r)$ and $G\overline{vc}2(=2\overline{wc}W/r)$], and the generation terms of $\overline{v^2}$, $\overline{uw}$ and $\overline{wc}$ ($G\overline{v^2}$, $G\overline{uw}$ and $G\overline{wc}$) are expressed in equations (4), (9) and (10), respectively. $G\overline{v^2}$, $G\overline{uw}$ and $G\overline{wc}$ are related to $\partial(rW)/\partial r$ and $\overline{vw}$, whose generation term is $G\overline{vw}=-(\overline{v^2}/r)\,\partial(rW)/\partial r+2\overline{w^2}W/r$. $\partial(rW)/\partial r$ and the generation term of $\overline{vw}$ involve swirl velocity W, and the quantities, $\overline{vw}$ and $\partial(rW)/\partial r$, introduce the swirl effects directly and exert effects on the generation terms of $\overline{v^2}$, $\overline{uw}$ and $\overline{wc}$, as well as on the generation terms of $\overline{uv}$ and $\overline{vc}$. Therefore, we propose that $\overline{vw}$ and $\partial(rW)/\partial r$ are the parameters which determine the swirl effects.

When the sign of $\partial(rW)/\partial r$ is negative ($\partial(rW)/\partial r \leqslant 0$), the generation term of $\overline{vw}$, $G\overline{vw}(=-(\overline{v^2}/r)\,\partial(rW)/\partial r+2\overline{w^2}W/r)$, is positive and $\overline{vw}$ is estimated to be positive on the basis of assumption (1). When the sign of $\partial(rW)/\partial r$ is positive ($\partial(rW)/\partial r > 0$), the generation term $G\overline{vw}$ can take either the positive or negative sign. Therefore, classification can be made into the following three cases according to the sign of both $\partial(rW)/\partial r$ and $\overline{vw}$:

Case 1 $\partial(rW)/\partial r \leqslant 0$ and $\overline{vw} > 0$

Case 2 $\partial(rW)/\partial r > 0$ and $\overline{vw} > 0$

Case 3 $\partial(rW)/\partial r > 0$ and $\overline{vw} \leqslant 0$.

2.2. *Cases* 1, 2 *and* 3

The swirl effect on the gradient-type generation terms [$G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$, $G\overline{vc}1(=-\overline{v^2}\,\partial C/\partial r)$] and on the other generation terms [$G\overline{uv}2(=2\overline{uw}W/r)$, $G\overline{vc}2(=2\overline{wc}W/r)$] is determined by criteria (1) and (2), respectively:

(1) If $\overline{v^2}$ is increased due to swirl, the gradient-type generation terms $G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$ and $G\overline{vc}1(=-\overline{v^2}\,\partial C/\partial r)$ increase and the gradient diffusions are promoted by swirl. On the other hand, decrease in $\overline{v^2}$ resulting from swirl induces retardation of gradient diffusion.

(2) The swirl effect related to the generation term $G\overline{uv}2(=2\overline{uw}W/r)$ is determined by whether $G\overline{uv}2$ has the same sign as $-\partial U/\partial r$ or not. The sign of $-\partial U/\partial r$ is identical to the sign of the gradient-type generation term $G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$. If $G\overline{uv}2(=2\overline{uw}W/r)$ and $-\partial U/\partial r$ have the same sign, $G\overline{uv}2$ has the effect of

promoting gradient diffusion, whereas the opposite sign induces suppression. The swirl effect related to the generation term $G\overline{vc}2(=2\overline{wc}W/r)$ is also classified in the same manner. If $G\overline{vc}2$ and $-\partial C/\partial r$ have the same sign, $G\overline{vc}2$ has the effect of promoting gradient diffusion, whereas the opposite sign induces suppression.

The mechanism of swirl effects in cases 1–3 are classified and summarized in Fig. 1(a)–(c) for momentum transport ($\overline{uv}$) and in Fig. 2(a)–(c) for scalar transport ($\overline{vc}$), respectively. The arrows with two types of lines, → and ⇒, represent swirl having promotion and suppression effects, respectively. The arrow ····> denotes that the swirl can take either the promotion or suppression effect.

Cases 1–3 are further discussed below.

Case 1. In case 1, where $\partial(rW)/\partial r \leqslant 0$ and $\overline{vw} > 0$ [Figs. 1(a) and 2(a)], we describe the effect of swirl on the generation terms of $\overline{uv}$, $G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$ and $G\overline{uv}2(=2\overline{uw}W/r)$, separately, and then combine them. After that, the effect of swirl on generation terms of $\overline{vc}$, $G\overline{vc}1(=-\overline{v^2}\,\partial C/\partial r)$ and $G\overline{vc}2(=2\overline{wc}W/r)$ is described.

In a nonswirling flow, the generation term does not appear in the $\overline{v^2}$ equation and $\overline{v^2}$ is redistributed from $\overline{u^2}$. In a swirling flow with positive $\overline{vw}$ as in case 1, $\overline{v^2}$ is increased due to its positive generation term $G\overline{v^2}(=4\overline{vw}W/r)$. This increases the generation term $G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$ and the gradient diffusion of momentum is enhanced.

$\overline{uw}$ is generated by the two generation terms, $G\overline{uw}1$ and $G\overline{uw}2$, in a swirling flow. The generation term $G\overline{uw}1(=-\overline{vw}\,\partial U/\partial r)$ has the same sign as $-\partial U/\partial r$ due to positive $\overline{vw}$. The other generation term $G\overline{uw}2[=-(\overline{uv}/r)\,\partial(rW)/\partial r]$ also has the same sign as $-\partial U/\partial r$ resulting from the conditions of $\overline{uv}(-\partial U/\partial r) > 0$ [equation (7)] and $\partial(rW)/\partial r \leqslant 0$ in case 1. Therefore, both of the generation terms of $\overline{uw}$, $G\overline{uw}1$ and $G\overline{uw}2$, cause $\overline{uw}$ to have the same sign as $-\partial U/\partial r$, and $G\overline{uv}2$ has the effect of promoting the gradient diffusion.

Due to the mechanism described above, both of the generation terms of $\overline{uv}$, $G\overline{uv}1$ and $G\overline{uv}2$, promote gradient diffusion of momentum due to swirl in case 1.

The swirl effect on the generation terms of $\overline{vc}$, $G\overline{vc}1$ and $G\overline{vc}2$, in case 1 is determined in the same manner as described above. Increase of $\overline{v^2}$ due to swirl increases the gradient-type generation term $G\overline{vc}1(=-\overline{v^2}\,\partial C/\partial r)$. $\overline{wc}$ is generated by two generation terms $G\overline{wc}1(=-\overline{vw}\,\partial C/\partial r)$ and $G\overline{wc}2[=-(\overline{vc}/r)\,\partial(rW)/\partial r]$ in swirling flows, and three conditions $\partial(rW)/\partial r \leqslant 0$, $\overline{vw} > 0$ and $\overline{vc}(-\partial C/\partial r) > 0$ [equation (8)] cause both $G\overline{wc}1$ and $G\overline{wc}2$ to have the same sign as $-\partial C/\partial r$. Therefore, both $G\overline{wc}1$ and $G\overline{wc}2$ generate $\overline{wc}$, and consequently, $G\overline{vc}2(=2\overline{wc}W/r)$ has the same sign as $-\partial C/\partial r$. In case 1, both $G\overline{vc}1$ and $G\overline{vc}2$ promote the gradient diffusion of the scalar.

In case 1, where $\partial(rW)/\partial r \leqslant 0$ and $\overline{vw} > 0$, swirl has the effect of promoting the gradient diffusion of momentum and scalar radial turbulent transport.

Case 2. In case 2 where $\partial(rW)/\partial r > 0$ and $\overline{vw} > 0$ [Figs. 1(b) and 2(b)], swirl has the effect of promoting the gradient diffusion through the generation term $G\overline{uv}1$, whereas the other generation term, $G\overline{uv}2$, can exert either effect. Therefore, the swirl effect on $\overline{uv}$ cannot be determined uniquely. The same is true for $\overline{vc}$. The reason is as follows.

The effect of swirl on $G\overline{uv}1$ is the same as in case 1. Increase of $\overline{v^2}$ due to its positive generation term $G\overline{v^2}(=4\overline{vw}W/r)$ under the condition $\overline{vw} > 0$ induces the increase of the gradient-type generation term $G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$ and the gradient diffusion of momentum is promoted by $G\overline{uv}1$.

$\overline{uw}$ is generated by the two generation terms, $G\overline{uw}1$ and $G\overline{uw}2$. $G\overline{uw}1(=-\overline{vw}\,\partial U/\partial r)$ causes $\overline{uw}$ to have the same sign as $-\partial U/\partial r$ due to positive $\overline{vw}$, whereas the other generation term $G\overline{uw}2[=-(\overline{uv}/r)\partial(rW)/\partial r]$ causes $\overline{uw}$ to have the opposite sign to $-\partial U/\partial r$ under the conditions of $\overline{uv}(-\partial U/\partial r) > 0$ [equation (7)] and $\partial(rW)/\partial r > 0$. Therefore, $\overline{uw}$ has the same sign as the larger of the generation terms, $G\overline{uw}1$ and $G\overline{uw}2$.

If $G\overline{uw}1$ is larger than $G\overline{uw}2$, $G\overline{uv}2$ has the same sign as $-\partial U/\partial r$. Both generation terms $G\overline{uv}1$ and $G\overline{uv}2$ promote gradient diffusion due to swirl. If $G\overline{uw}2$ is larger than $G\overline{uw}1$, $G\overline{uv}2$ has an opposite sign to $G\overline{uv}1$. If $G\overline{uv}1$ is larger than $G\overline{uv}2$, swirl enhances the gradient diffusion, whereas if $G\overline{uv}1$ is smaller than $G\overline{uv}2$, swirl suppresses the gradient diffusion.

Swirl effects on the generation terms of $\overline{vc}$, $G\overline{vc}1(=-\overline{v^2}\,\partial C/\partial r)$ and $G\overline{vc}2(=2\overline{wc}W/r)$, in case 2 are determined in the same manner as described above. Increase of $\overline{v^2}$ due to swirl induces increase of the gradient-type generation term $G\overline{vc}1(=-\overline{v^2}\,\partial C/\partial r)$. $\overline{wc}$ is generated by the two generation terms $G\overline{wc}1$ and $G\overline{wc}2$ in swirling flows. Three conditions, $\partial(rW)/\partial r > 0$, $\overline{vw} > 0$ and $\overline{vc}(-\partial C/\partial r) > 0$ [equation (8)], cause $G\overline{wc}1$ and $G\overline{wc}2$ to have opposite signs. Therefore, $\overline{wc}$ has the same sign as the larger of the generation terms, $G\overline{wc}1$ or $G\overline{wc}2$. If $G\overline{wc}1$ is larger than $G\overline{wc}2$, $G\overline{vc}2$ has the same sign as $-\partial C/\partial r$. Both of the generation terms $G\overline{vc}1$ and $G\overline{vc}2$ promote gradient diffusion due to swirl. If $G\overline{wc}2$ is larger than $G\overline{wc}1$, $G\overline{vc}2$ has the opposite sign to $G\overline{vc}1$. If $G\overline{vc}1$ is larger than $G\overline{vc}2$, swirl enhances gradient diffusion, whereas if $G\overline{vc}1$ is smaller than $G\overline{vc}2$, swirl suppresses gradient diffusion.

Case 3. For case 3, where $\partial(rW)/\partial r > 0$ and $\overline{vw} \leqslant 0$ [Figs. 1(c) and 2(c)], we describe the swirl effects on the generation terms of $\overline{uv}$, $G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$ and $G\overline{uv}2(=2\overline{uw}W/r)$, separately, and then combine them. After that, effects of swirl on the generation terms of $\overline{vc}$, $G\overline{vc}1(=-\overline{v^2}\,\partial C/\partial r)$ and $G\overline{vc}2(=2\overline{wc}W/r)$, are described.

In swirling flow with negative $\overline{vw}$, $\overline{v^2}$ decreases due to the negative generation term $G\overline{v^2}(=4\overline{vw}W/r)$. This induces decrease of the generation term $G\overline{uv}1(=-\overline{v^2}\,\partial U/\partial r)$ and the gradient diffusion of momentum due to $G\overline{uv}1$ is suppressed.

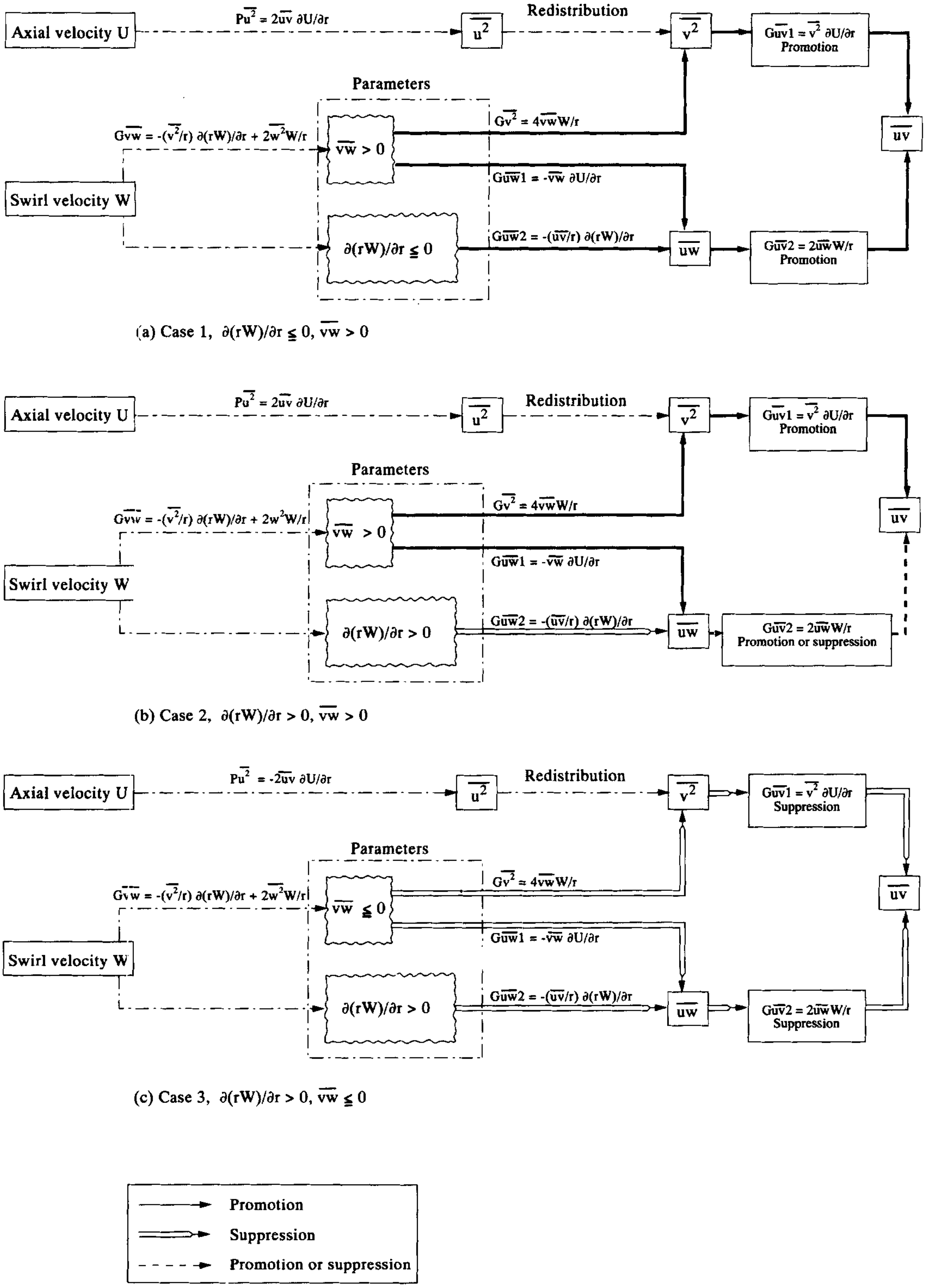

Fig. 1. The mechanism of swirl effects in cases 1–3 for momentum transport.

$\overline{uw}$ is generated by two generation terms, $G\overline{uw}1$ and $G\overline{uw}2$ due to swirl. The generation term $G\overline{uw}1(= -\overline{vw}\,\partial U/\partial r)$ has the opposite sign to $-\partial U/\partial r$ due to negative $\overline{vw}$. The other generation term $G\overline{uw}2[= -(\overline{uv}/r)\,\partial(rW)/\partial r]$ also has the opposite sign to $-\partial U/\partial r$ under the conditions of $\overline{uv}(-\partial U/\partial r) > 0$ [equation (7)] and $\partial(rW)/\partial r > 0$. Therefore, both generation terms of $\overline{uw}$, $G\overline{uw}1$ and $G\overline{uw}2$, cause $\overline{uw}$ to have the opposite sign to $-\partial U/\partial r$, and $G\overline{uv}2$ suppresses the gradient diffusion.

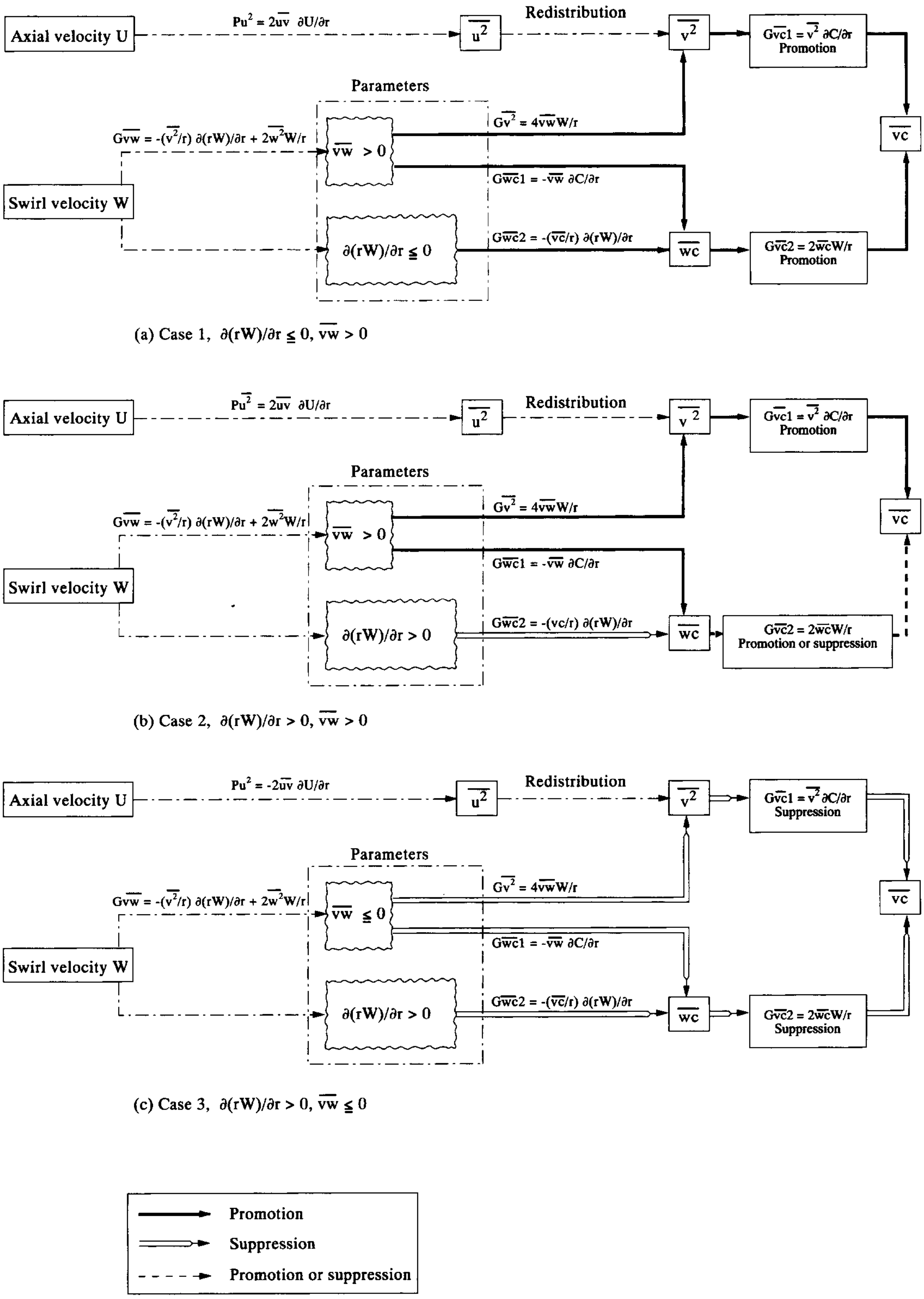

Fig. 2. The mechanism of swirl effects in cases 1–3 for scalar transport.

Due to the mechanism mentioned above, both generation terms of $\overline{uv}$, $G\overline{uv}1$ and $G\overline{uv}2$ suppress the gradient diffusion of momentum due to swirl in case 3.

Swirl effect on the generation terms of $\overline{vc}$, $G\overline{vc}1$ and $G\overline{vc}2$, in case 3 is determined in the same manner as described above. Decrease of $\overline{v^2}$ due to swirl decreases the gradient-type generation term $G\overline{vc}1(= -\overline{v^2}$

$\partial C/\partial r$). $\overline{wc}$ is generated by the two generation terms $G\overline{wc}1(= -\overline{vw}\partial C/\partial r)$ and $G\overline{wc}2[= -(\overline{vc}/r)\partial(rW)/\partial r]$ in swirling flows, and the three conditions $\partial(rW)/\partial r > 0$, $\overline{vw} \leqslant 0$ and $\overline{vc}(-\partial C/\partial r) > 0$ [equation (8)] cause both $G\overline{wc}1$ and $G\overline{wc}2$ to have the opposite sign to $-\partial C/\partial r$. Therefore, both $G\overline{wc}1$ and $G\overline{wc}2$ cause $\overline{wc}$, and consequently $G\overline{vc}2(= 2\overline{wc}W/r)$, to have the opposite sign to $-\partial C/\partial r$. In case 3, both $G\overline{vc}1$ and $G\overline{vc}2$ suppress gradient diffusion.

In case 3, where $\partial(rW)/\partial r \leqslant 0$ and $\overline{vw} > 0$, swirl retards the momentum and scalar gradient diffusion of radial transport.

2.3. *Countergradient diffusion*

When the suppression effect due to swirl on the turbulent transport becomes extreme, countergradient diffusion may be induced, which can be expressed as equations (11) and (12) for momentum and scalar transport, respectively:

$$\overline{uv}(-\partial U/\partial r) < 0 \quad (11)$$

$$\overline{vc}(-\partial C/\partial r) < 0. \quad (12)$$

We will describe the condition required for the countergradient diffusion of momentum to occur, and then describe the scalar case.

The generation term $G\overline{uv}1(= -\overline{v^2}\,\partial U/\partial r)$ always drives the gradient diffusion of momentum. If the countergradient diffusion occurs, the other additional generation term $G\overline{uv}2(= 2\overline{uw}W/r)$ including swirl velocity W should have a large suppression effect on $\overline{uv}$. $\overline{uw}$ in $G\overline{uv}2$ is generated by $G\overline{uw}1$ and $G\overline{uw}2$. Therefore, $G\overline{uv}2$ has a suppression effect in the following two cases.

The first case is when $\overline{vw} < 0$. This condition induces $G\overline{uw}1(= -\overline{vw}\,\partial U/\partial r)$ to have the opposite sign to $-\partial U/\partial r$. When the effect of $G\overline{uw}1$ on $\overline{uw}$ is large, $G\overline{uv}2(= 2\overline{uw}W/r)$ has the opposite sign to $-\partial U/\partial r$ and suppresses $\overline{uv}$.

The second case is when $\partial(rW)/\partial r < 0$. The conditions of $\overline{uv}(-\partial U/\partial r) < 0$ [equation (11)] and $\partial(rW)/\partial r < 0$ dictate that $G\overline{uw}2[= -(\overline{uv}/r)\,\partial(rW)/\partial r]$ has the opposite sign to $-\partial U/\partial r$. When the effect of $G\overline{uw}2$ on $\overline{uw}$ is large, $G\overline{uv}2(= 2\overline{uw}W/r)$, including $\overline{uw}$, has the opposite sign to $-\partial U/\partial r$ and suppresses $\overline{uv}$.

$G\overline{uv}2$ has the effect of suppression in the two cases mentioned above. When the effect of $G\overline{uv}2$ on $\overline{uv}$ is larger than the promotion effect of $G\overline{uv}1$, countergradient diffusion of momentum occurs.

The generation term $G\overline{vc}1(= -\overline{v^2}\,\partial C/\partial r)$ always has the effect of driving scalar gradient diffusion. If a scalar countergradient diffusion occurs, the other additional generation term $G\overline{vc}2(= 2\overline{wc}W/r)$ including swirl velocity should have a large suppression effect on $\overline{vc}$. $\overline{wc}$ in $G\overline{vc}2$ is generated by $G\overline{wc}1$ and $G\overline{wc}2$. Therefore, $G\overline{vc}2$ has a suppression effect in the following two cases.

The first case is when $\overline{vw} < 0$. This condition induces $G\overline{wc}1(= -\overline{vw}\,\partial C/\partial r)$ to have the opposite sign to $-\partial C/\partial r$, and when the effect of $G\overline{wc}1$ on $\overline{wc}$ is large, $G\overline{vc}2(= 2\overline{wc}W/r)$ has the opposite sign to $-\partial C/\partial r$ and suppresses $\overline{vc}$.

The second case is when $\partial(rW)/\partial r < 0$. The conditions of $\overline{vc}(-\partial C/\partial r) < 0$ [equation (12)] and $\partial(rW)/\partial r < 0$ dictate that $G\overline{wc}2[= -(\overline{vc}/r)\,\partial(rW)/\partial r]$ has the opposite sign to $-\partial C/\partial r$. When the effect of $G\overline{wc}2$ on $\overline{wc}$ is large, $G\overline{vc}2(= 2\overline{wc}W/r)$ has the opposite sign to $-\partial C/\partial r$ and suppresses $\overline{vc}$.

$G\overline{vc}2$ has the effect of suppression in the above two cases. When the suppression effect of $G\overline{vc}2$ is larger than the promotion effect of $G\overline{vc}1$, scalar countergradient diffusion occurs.

3. CONCLUSIONS

Profiles of axial velocity U and scalar C in an axisymmetric boundary-layer-type swirling flow are dominated by radial turbulent transport $\overline{uv}$ and $\overline{vc}$, respectively. Swirl effects on $\overline{uv}$ and $\overline{vc}$ are strongly related to their generation terms $G\overline{uv}1(= -\overline{v^2}\,\partial U/\partial r)$, $G\overline{uv}2(= 2\overline{uw}W/r)$, $G\overline{vc}1(= -\overline{v^2}\,\partial C/\partial r)$ and $G\overline{vc}2$ $(= 2\overline{wc}W/r)$. $\overline{v^2}$, $\overline{uw}$ and $\overline{wc}$ included in them are directly generated by the swirling flow. Through careful consideration of the relationships between the generation terms of $\overline{v^2}$, $\overline{uw}$ and $\overline{wc}$ and swirl, parameters $\overline{vw}$ and $\partial(rW)/\partial r$, which govern whether swirl promotes or suppresses the gradient diffusion, are derived. In the case of $\partial(rW)/\partial r \leqslant 0$ and $\overline{vw} > 0$, swirl promotes turbulent transport, whereas in the case of $\partial(rW)/\partial r > 0$ and $\overline{vw} \leqslant 0$, it suppresses turbulent transport. In the case of $\partial(rW)/\partial r > 0$ and $\overline{vw} > 0$, swirl can affect either of the two cases. These conclusions are valid where countergradient diffusion does not occur. Countergradient diffusion can occur under the condition of $\overline{vw} < 0$ or $\partial(rW)/\partial r < 0$, where the suppression effect of $G\overline{uv}2$ and $G\overline{vc}2$ overcomes the promotion effect of $G\overline{uv}1$ and $G\overline{vc}1$, respectively.

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